Pion distribution amplitude – from theory to data

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Abstract. We describe the present status of the pion distribution amplitude as originated from two sources: (i) a nonperturbative approach, based on QCD sum rules with nonlocal condensates, and (ii) a NLO QCD analysis of the CLEO data on $F^{\gamma\gamma^*\pi}(Q^2)$, supplemented by the E791 data on diffractive dijet production, and the JLab F(pi) data on the pion electromagnetic form factor.

1. PION DISTRIBUTION AMPLITUDE FROM QCD SUM RULES

The pion distribution amplitude (DA) can be defined through the matrix element of a nonlocal axial current on the light cone

$$\langle 0 \mid \bar{d}(z)\gamma_\mu\gamma_5\mathscr{C}(z,0)u(0) \mid \pi(P)\rangle\Big|_{z^2=0} \;=\; i f_\pi P_\mu \int_0^1 dx\, e^{ix(zP)}\, \varphi_\pi(x,\mu^2)\,, \qquad (1)$$

which is explicitly gauge-invariant due to the connector $\mathscr{C}(z,0) = \mathscr{P}e^{ig\int_0^z A_\mu(\tau)d\tau^\mu}$. This amplitude describes the transition of the physical pion $\pi(P)$ to a pair of valence quarks u and d, separated on the light-cone, with corresponding momentum fractions xP and $\bar{x}P$, (we set $\bar{x} \equiv 1-x$).

In order to obtain the pion DA theoretically we use, following Mikhailov and Radyushkin [1], a QCD sum rule approach with non-local condensates (NLC). Just for illustration, we present here the simplest scalar condensate of the used NLC model, which reads

$$\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle\, e^{-|z^2|\lambda_q^2/8}\,. \qquad (2)$$

This model is determined by a single scale parameter $\lambda_q^2 = \langle k^2\rangle$ characterizing the average momentum of quarks in the QCD vacuum. It has been estimated in QCD SRs [2, 3] and on the lattice [4, 5]: $\lambda_q^2 = 0.45 \pm 0.1$ GeV^2.

The NLC sum rules for the pion DA produce [6] a "**bunch**" of self-consistent 2-parameter models at $\mu^2 \simeq 1$ GeV^2:

$$\varphi_\pi(x) = \varphi^{\text{as}}(x)\left[1 + a_2 C_2^{3/2}(2x-1) + a_4 C_4^{3/2}(2x-1)\right]. \qquad (3)$$

By self-consistency we mean that the value of the inverse moment for the whole "bunch" $\langle x^{-1}\rangle_\pi^{\text{bunch}} = 3.17 \pm 0.10$ is in agreement with an independent estimate from another sum rule, viz., $\langle x^{-1}\rangle_\pi^{\text{SR}} = 3.30 \pm 0.30$. For the favored value $\lambda_q^2 = 0.4$ GeV^2, we get the "bunch" of pion DAs presented in Fig. 1a. We also extracted the corresponding "bunches" for two other values of $\lambda_q^2 = 0.5$ GeV^2 and $\lambda_q^2 = 0.6$ GeV^2, and show the results (rectangle areas) in the (a_2, a_4)-plane in Fig. 1b.

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[1] Talk based on results obtained in collaboration with S. Mikhailov, K. Passek-Kumerički, W. Schroers, and N. G. Stefanis

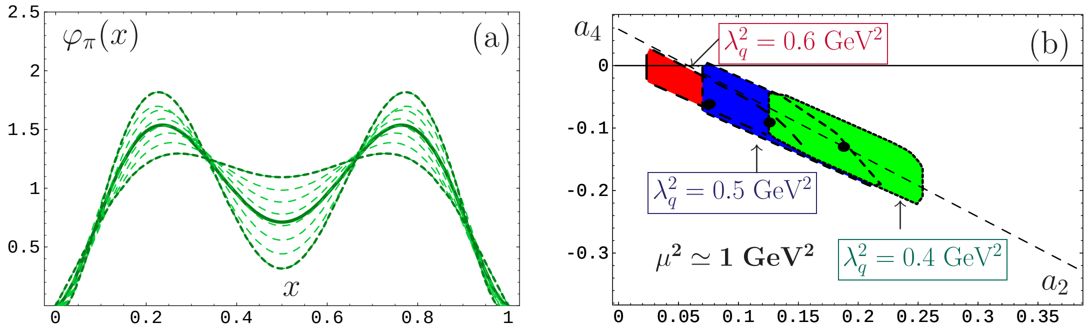

FIGURE 1. (a) The bunch of pion DAs extracted from NLC QCD sum rules. The parameters of the bold-faced curve are $a_2^{\mathbf{b.f.}} = +0.188$ and $a_4^{\mathbf{b.f.}} = -0.130$. (b) The "bunches" of pion DAs extracted from NLC QCD sum rules in the (a_2, a_4)-plane for three values of the nonlocality parameter λ_q^2.

2. NLO LIGHT-CONE SUM RULES (LCSR) AND THE CLEO DATA

The CLEO experimental data on $F^{\gamma\gamma^*\pi}(Q^2)$ allow one to obtain direct constraints on $\varphi_\pi(x)$. For $Q^2 \gg m_\rho^2$, $q^2 \ll m_\rho^2$ pQCD factorization is valid only in leading twist, but higher twists are also of importance [7]. The reason is: if $q^2 \to 0$ one needs to take into account the interaction of a real photon at long distances of order of $O(1/\sqrt{q^2})$. Applying the LCSR approach [8], one effectively accounts for the long-distance effects of a real photon, using the quark-hadron duality in the vector channel and a dispersion relation in q^2.

In our CLEO data analysis [13], we also took into account the relation between the "nonlocality" scale and the twist-4 magnitude $\delta^2_{\text{Tw-4}} \approx \lambda_q^2/2$, which was used to re-estimate $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.02$ at $\lambda_q^2 = 0.4$ GeV^2. To make our conclusions more precise, we have adopted a 20% uncertainty in the magnitude of the twist-4 contribution, $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ GeV^2, and produced new 1σ-, 2σ- and 3σ-contours dictated by the CLEO data [14]. We concluded that even with a 20% uncertainty in $\delta^2_{\text{Tw-4}}$, the CZ DA is excluded **at least** at the 4σ-level, whereas the asymptotic DA – at the 3σ-level. Our "bunch" is inside the 1σ-region and other nonperturbative models are near the 3σ-boundary. We show in Fig. 2a the plot of $Q^2 F_{\gamma^*\gamma\to\pi}(Q^2)$ for our "bunch" (shaded strip), CZ DA (upper dashed line), asymptotic DA (lower dashed line), and two instanton-based models (dotted [15] and dash-dotted [16] lines) in comparison with the CELLO and CLEO data. We see that the BMS "bunch" describes rather well all data for $Q^2 \gtrsim 1.5$ GeV^2.

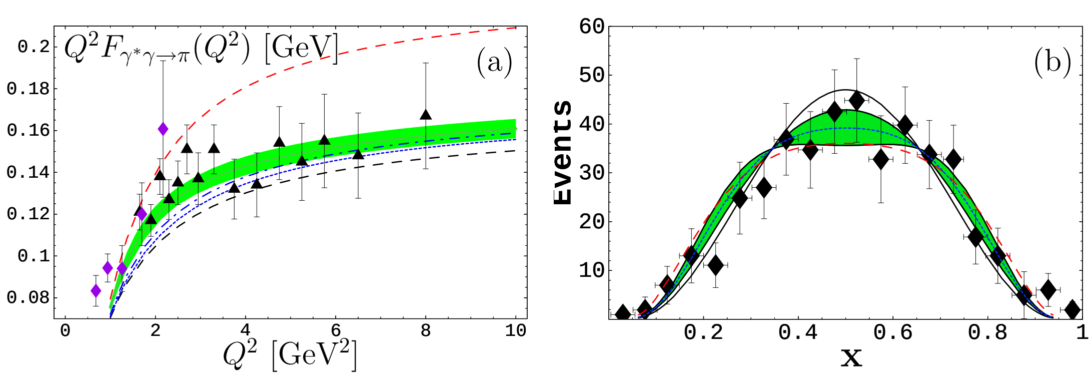

FIGURE 2. (a) $\gamma^*\gamma \to \pi$ Transition form factor in comparison with the CELLO (◆) [9] and the CLEO (▲) [10] data. For details see in the text. (b) Comparison of the asymptotic DA (solid line), Chernyak–Zhitnitsky (CZ) DA (dashed line) [11], and the BMS "bunch" of pion DAs (strip) with the E791 data (◆) [12].

3. DIFFRACTIVE DIJET PRODUCTION

The diffractive dijet production in $\pi + A$ collisions has been suggested as a tool to extract the profile of the pion DA by Frankfurt et al. in 1993 [17]. They argued that the jet distribution with respect to the longitudinal momentum fraction has to follow the quark momentum distribution in the pion and hence provides a direct measurement of the pion DA. As it was shown just recently in [18] (see also [19]), this proportionality does not hold beyond the leading logarithms in energy. Braun et al. found that the distribution in the longitudinal momentum fraction of the jets for the non-factorizable contribution is the same as for the factorizable contribution with the asymptotic pion DA. Using the convolution approach of Braun et al. we estimated [14] the distribution of jets in this experiment for our "bunch" of pion DAs and show the results in comparison with $\varphi^{\rm as}$ and $\varphi^{\rm CZ}$ in Fig. 2b. It is interesting to note that the corresponding χ^2 values are: as – 12.56; CZ – 14.15; BMS – 10.96 (accounting for 18 data points). The main conclusion from this comparison: **all three DAs are compatible with the E791 data**. Hence, this experiment cannot serve as a safe profile indicator.

Let us say a few words about similarities and differences between the CZ and BMS DAs. Both are two-humped, but the CZ DA is strongly end-point enhanced, whereas the BMS DA is end-point suppressed, as is well illustrated in Fig. 3a. And the reason for this behavior is physically evident: nonlocal quark condensate reduces pion DA in the small x region and enhances in the vicinity of the point $x \simeq 0.2$. In order to keep the norm equal to unity, it is forced to have in the central region some reduction as well.

4. PION ELECTROMAGNETIC FORM FACTOR

How well is the BMS bunch in comparison with the JLab data on the pion form factor? We have calculated the pion form factor in analytic NLO pQCD [20]

$$F_\pi(Q^2;\mu_{\rm R}^2) = F_\pi^{\rm LD}(Q^2) + \left(\frac{Q^2}{2s_0^{\text{2-loop}} + Q^2}\right)^2 F_\pi^{\rm Fact}(Q^2;\mu_{\rm R}^2)\,, \tag{4}$$

taking into account the soft part $F_\pi^{\rm LD}(Q^2)$ via the local duality approach [21], based on perturbative spectral density $\rho(s_1,s_2,Q^2)$ [22, 21], and correcting the factorized contribution $F_\pi^{\rm Fact}$ via

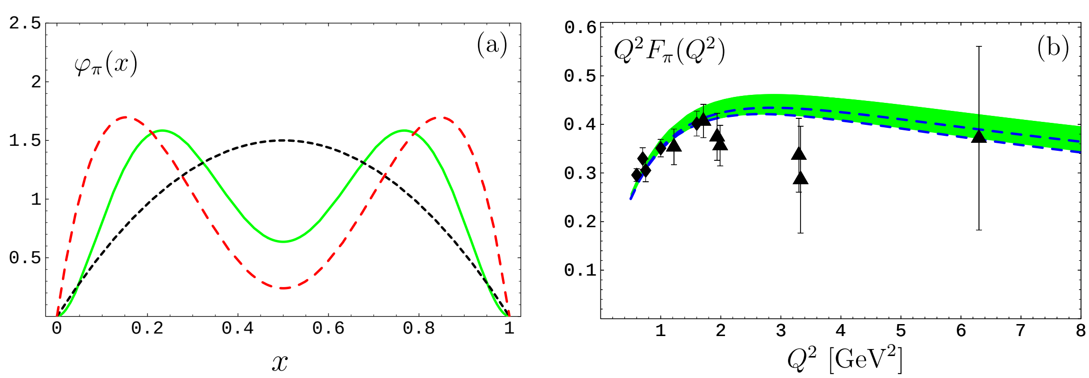

FIGURE 3. (a) Comparison of $\varphi^{\rm as}$, (dotted line), $\varphi^{\rm CZ}$ (dashed line), and $\varphi^{\rm BMS}$ (solid line). (b) Pion electromagnetic form factor in comparison with the JLab (◆) [23] and Bebek et al. (▲) [24] data. Predictions based on the BMS "bunch" of pion DAs (green strip) including the NLC QCD sum-rule uncertainties and those due to scale-setting ambiguities at the NLO level. The region between the dashed lines denotes the area accessible to $\varphi^{\rm as}$.

a power-behaved pre-factor (with $s_0^{\text{2-loop}} \approx 0.6\ \text{GeV}^2$) in order to respect the Ward identity at $Q^2 = 0$ while preserving its high-Q^2 asymptotics.

In our analysis $F_\pi^{\text{Fact}}(Q^2;\mu_{\text{R}}^2)$ has been computed to NLO [25, 26] using Analytic Perturbation Theory [27, 28, 29] and trading the running coupling and its powers for analytic expressions in a non-power series expansion, i.e.,

$$\left[F_\pi^{\text{Fact}}(Q^2;\mu_{\text{R}}^2)\right]_{\text{MaxAn}} = \bar{\alpha}_{\text{s}}^{(2)}(\mu_{\text{R}}^2)\,\mathscr{F}_\pi^{\text{LO}}(Q^2) + \frac{1}{\pi}\mathscr{A}_2^{(2)}(\mu_{\text{R}}^2)\,\mathscr{F}_\pi^{\text{NLO}}(Q^2;\mu_{\text{R}}^2)\,, \tag{5}$$

with $\bar{\alpha}_{\text{s}}^{(2)}$ and $\mathscr{A}_2^{(2)}(\mu_{\text{R}}^2)$ being the 2-loop analytic images of $\alpha_{\text{s}}(Q^2)$ and $\left(\alpha_{\text{s}}(Q^2)\right)^2$, respectively [20], whereas $\mathscr{F}_\pi^{\text{LO}}(Q^2)$ and $\mathscr{F}_\pi^{\text{NLO}}(Q^2;\mu_{\text{R}}^2)$ are the LO and NLO parts of the factorized form factor. This procedure with the analytic running coupling and the analytic versions of its powers gives us a practical independence of the scheme/scale setting and provides results in rather good agreement with the experimental data [24, 23]. Indeed, the form-factor predictions are only slightly larger than those resulting from the calculation with the asymptotic DA (see Fig. 3b).

5. CONCLUSIONS

The QCD sum rule method with nonlocal condensates produces a "bunch" of admissible pion DAs for each λ_q^2 value. Comparing these results with the CLEO constraints, obtained in the LCSR analysis of the $\gamma^*\gamma \to \pi$-transition form factor, clearly fixes the value of QCD vacuum nonlocality to $\lambda_q^2 = 0.4\ \text{GeV}^2$. The corresponding "bunch" of pion DAs agrees well with both the E791 data on diffractive dijet production and with the JLab F(pi) data on the pion electromagnetic form factor. Analytic perturbation theory with a non-power NLO contribution for the pion form factor diminishes scale-setting ambiguities already at the NLO level.

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